Solving with or without equations

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Plan of the talk

The **pentahedron** problem shows the proximity btw Geometric Theorem Proving and Geometric Constraint Solving

The two fields separate: **specificities of GCS**, which goes **from equations to algorithms**.

GCS examples in CADCAM.

GCS still benefits from symbolic tools, like **DAG**, and **dual numbers**.

From algorithms to equations: what if algorithms were just user-friendly way to pose equations, after all?

Pentahedron problem

Many common issues in both GTP & GCS:

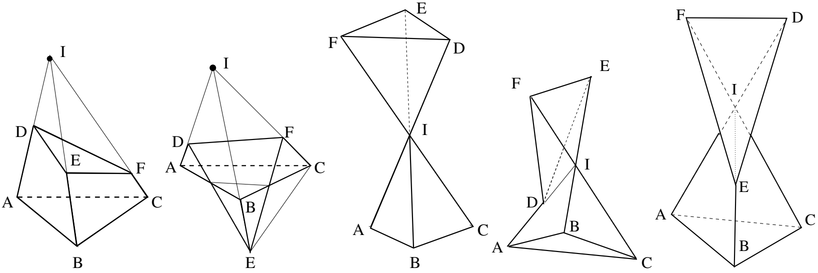
- dimension of the solution manifold,

- manifolds of spurious (degenerate) roots,

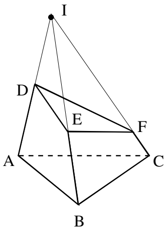
- points at infinity,

- many ways to pose equations.

Pentahedron problem

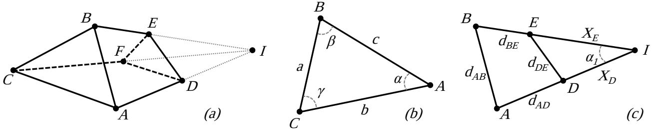


Pentahedron problem



First formulation: fix *ABC* in Oxy ⇒ 9 unknowns and equations: coplanarities of 3 quadrilateral faces and 6 pt-pt distances.

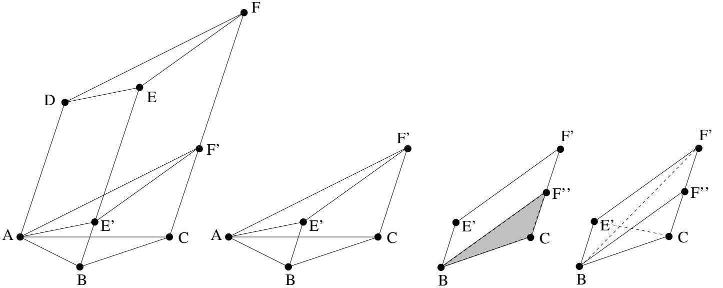
Pentahedron problem: I is not at infinity



$$\cos\alpha = \frac{x_D^2 + x_E^2 - d_{DE}^2}{2x_D x_E} = \frac{(x_D + d_{AD})^2 + (x_E + d_{BE})^2 - d_{AB}^2}{2(x_D + d_{AD})(x_E + d_{BE})}$$

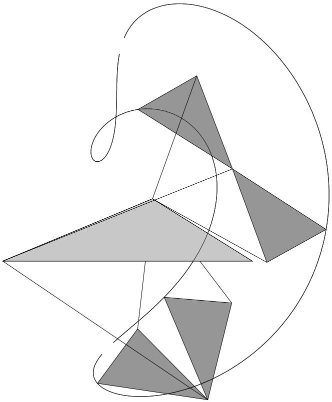
Better **coordinates-free** formulation, 3 times smaller. 40 times faster to solve with intervals. 3 unknowns are lengths ID, IE, IF. 3 relations for angle at I.

Pentahedron pbm: parallel solutions



There is almost always parallel solutions! and an easy geometric construction.

Pentahedron pbm: spurious roots, flat pentahedra



There is a finite number of spurious roots: **flat pentahedra**. Pin *ABC*, forget constraint *CF*: *DEF* can move around *ABC*. At most 6 roots (intersection between sextic curve and circle).

Pentahedron problem, manifold of spurious roots

Hexaedron or dodecahedron: a manifold of flat solutions.

Known difficulty in GTP: specify non degeneracy conditions in order to prove geometric theorems

Numerical analysis: deflation methods

Interval Analysis: the problem seems less known. Hint: search the root closest to a given point: the solution set is discrete.

The pentahedron problem

The pentahedron problem shows that:

GCS and GTP are very close while all constraints are:

incidence / distance / angle constraints between points / lines /planes.

But it is not sufficient for CADCAM, and GCS have **specificities**.

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Specificities: 1, Inaccuracy issue

Even with the simplest constraints, there are issues / troubles specific to GCS:

Inaccuracy issue

$\Rightarrow$ hard to compute the rank of Jacobians

$\Rightarrow$ distinction between $x > 0$ and $x \geq 0$ is irrelevant

$\Rightarrow$ the trick $x \neq 0 \Leftrightarrow xy - 1 = 0$ (where y is auxiliary) used in Grobner Bases makes no sense